**RANK OF A MATRIX**

**Definition of rank of a matrix**: The number **r** is called the rank of matrix **A** if

1. Every minor of order (r+1) of A is zero and
2. There exist at least one minor of order **r** which is non- zero.

**Note:**

1. The rank of a matrix A is denoted by .
2. The rank of a identity (or unit) matrix of order n is always equal to n.
3. The rank of a matrix **A** is equal to its transpose matrix i.e. = .
4. The rank of a non-singular matrix of order n is equal to n.
5. The rank of a zero (null) matrix is zero.

For example: (1) Let A= 

Then |A|= =0

Again 

Thus there exist at least one minor of order 2 of the matrix A which is non zero.

 = 2

**Elementary Row Transformations:**

The following three types of operations on the rows of a given matrix are known as **elementary row transformations** (i.e. E-row transformation ).

1. Interchanging any two rows of the given matrix (e.g. interchange of ith and jth rows will be denoted by RiRj ).
2. Multiplying every element of any row of the given matrix by a non-zero number say k (e.g. the multiplying of the ith row by a non-zero number say k will be denoted by Ri kRi).
3. Adding a non-zero scalar multiple of the elements of any row to the corresponding elements of another row (e.g. the addition of k times the jth row to the ith row will be denoted by Ri  Ri + kRj ).

Similarly we can have **elementary column transformations** (i.e. E-column transformation). The corresponding column transformation will be denoted by C, in place of R i.e. CiCj, Ci kCi, Ci  Ci + kCj respectively.

**Note(1).** The rank of a matrix remains unchanged by the application of any of the row (or column) transformation on it.

**Elementary Operations:** The above six operations (i) three elementary row operations and(ii) three elementary column operations, are called **Elementary Operations.**

**Definition of Row Equivalent:** A matrix **A** is said to be row equivalent to a matrix **B** if **B** can be obtained from **A** by applying in succession a finite number of elementary row operations on **A** and we write .

**Definition Column Equivalent:** A matrix **A** is said to be column equivalent to a matrix **B** if **B** can be obtained from **A** by applying in succession a finite number of elementary column operations on **A** and we write .

**Note.** Row- equivalent or Column- equivalent matrices have the same order and the same rank.

**Echelon Form of a Matrix:** A matrix is said to be in **Echelon form** if

1. all the zero rows occur below non-zero rows.
2. the number of zero before the first non-zero element in a row is less than the number of such zeros in the next row.
3. the first non-zeros element in every non-zero row is 1.

For example, the matrices

,  and 

are in **Echelon form**.

**Important Note:**

1. The number of non-zero rows in the Echelon form of a matrix is the rank of the given matrix.
2. A zero matrix and an identity matrix are always in row reduced Echelon form.

**Question**: Find the rank of the following matrices by reducing it to Echelon form:

1. A=  (ii) A= 
2. A= 

**Solution:**

1. Given that,

A= 

~  (R2R2-2R1)

~  (R1R2)

~  (R1 -1R1)

~  (R24R1)

~  (R2-R2)

Which is the Echelon form of the given matrix.

Since the number of non-zero row is two, therefore its rank is 2.

1. Given that,

A= 

~  (R2R2-3R1 & R3R3-R1)

~  (R3R3-R2)

~  ( R2-R2 )

Which is the Echelon form of the given matrix.

Since the number of non-zero row is two, therefore its rank is 2.

1. Given that,

A= 

~  (R2R2+2R1 ; R3R3-R1)

~  (R2R2 ; R3-R3)

~  (R3R3-R2 ; R4R4-R2)

Which is the Echelon form of the given matrix.

Since the number of non-zero row is two, therefore its rank is 2.

**Normal Form of a Matrix:**

We know that successive application of elementary row operations on a matrix reduce it to the row Echelon form. Now if we apply both elementary row and column operations, we shall get a still simple matrix called **normal or canonical form** of the matrix.

If we apply successively elementary operations, we can reduce any matrix **A** of rank  in one of the following forms:

 ,  ,  , 

( where  is the identity matrix of order **r** and O is some row matrix)

All these are known as normal form of the matrix **A**.

**Note:**

1. If r =0, then **A** is the normal form if and only if **A** = 0.
2. If **A =**  or  or  or  then =r.
3. If **A** is an matrix of rank r, then **A** is equivalent to the matrix  in the normal form.

**Question:** Find the rankof the matrix

A= 

by reducing it to normal form.

**Solution:** Given that,

A= 

~ (R1R1 , R2R2 )

~ (R2R2-2R1, R3R3-R1, R4R4-R1)

~  (C2C2+C1, C4C4-3C1)

~  (R2R4)

~  ( R2(-1)R1 )

~  (R4R4-3R2)

~  (C3C3+C2, C4C4-C2)

~  (R1R3)

~  (C3C3 , C4-C4 )

~  (C4C4-C3)

Which is the normal form . Hence rank of the matrix **A =** 3.