Now $AX = B$

$\Rightarrow X = A^{-1} B.$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/18 & -1/18 & 1/18 \\ 1/3 & 1/3 & -19/36 \\ -1/3 & 5/36 & 13/36 \end{bmatrix} \begin{bmatrix} 1 \\ 52 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1/18 - \frac{52}{18} + \frac{4}{18} \\ \frac{1}{3} + \frac{52}{3} - \frac{76}{36} \\ -\frac{1}{3} + \frac{260}{36} + \frac{52}{36} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3-26+2}{9} \\ \frac{3+13-19}{9} \\ \frac{-3+65+13}{9} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7/3 \\ -1/3 \\ -25/3 \end{bmatrix}$$

$\therefore x = -7/3 \Rightarrow 3x = -7$

$y = -1/3 \Rightarrow 3y = -1$

$z = -25/3 \Rightarrow 3z = -25$

which is the required solution of the given equation. #

10/10/09

(iii) The given system of equations are

$$x+y+z=1 \quad \text{---(1)}$$
$$3x+4y+5z=2 \quad \text{---(2)}$$
$$2x+3y+4z=1. \quad \text{---(3)}$$

The above equation can be written in matrix form as

$$AX=B \quad \text{---(A)}$$

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix}$

$$= 1(16-15) - 1(12-10) + 1(9-8)$$
$$= 1-2+1$$
$$= 0.$$

Since $|A|=0$ therefore the given system of eqation has no unique solution.

Let us find adj A.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

1st row:-

Co-factor of $1 = \begin{vmatrix} 4 & 5 \\ 3 & 4 \end{vmatrix} = 1$

" " $1 = -\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} = -2$

" " $1 = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = 1.$

2nd row:-

Co-factor of $3 = -\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = -1$

" " $4 = \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 2$

" " $5 = -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1.$

3rd row:-

Co-factor of $2 = \begin{vmatrix} 1 & 1 \\ 4 & 5 \end{vmatrix} = 1.$

" " $3 = -\begin{vmatrix} 1 & 1 \\ 3 & 5 \end{vmatrix} = -2$

" " $4 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1.$

$\therefore$ Co-factor matrix of $A = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 1 \end{bmatrix}$

$$\therefore \operatorname{adj} A = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 2 & -2 \\ 1 & -1 & 1 \end{bmatrix}$$

We now have,

$$(\operatorname{adj} A)B = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 2 & -2 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+1 \\ -2+4-2 \\ 1-2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$, which is a zero matrix.

Since, $|A|=0$ & $(\operatorname{adj} A)B$ is a zero matrix, Therefor the given system of equations has infinite no. of solutions.

Let us take $z=k$. Then from (1) & (2) we have -

$x+y+k=1 \Rightarrow x+y=1-k \longrightarrow (4)$

& $3x+4y+5k=2 \Rightarrow 3x+4y=2-5k \longrightarrow (5)$

$(5)-(4)\times(3) \Rightarrow y=-1-2k$

$\therefore (4) \Rightarrow x-1-2k=1-k$

$\Rightarrow x = 2+k$