01. Q:→ Prove that the set G consisting of the fourth roots of unity is an abelian group under multiplication of complex numbers. Is G is a cyclic group? Justify your answer.

Proof:→ Given that,

G = The set of fourth roots of unity.
$= \{1, -1, i, -i\}$.

We now show that G is an abelian group w.r.to multiplication as binary operation.

(I) Clearly product of any two elt. of G is also an elt. of G.

(II) Again associative law hold good in G being a subset of a complex number.

(III) Clearly $1 \in G$, which is the identity elt. of G.

(IV) We ~~have~~ have,

$$1^{-1} = 1 \in G$$
$$(-1)^{-1} = \frac{1}{-1} = -1 \in G$$
$$i^{-1} = \frac{1}{i} = -i \in G$$
$$(-i)^{-1} = \frac{1}{-i} = \frac{i}{-i^2} = i \in G$$

This shows that every elt. of G has its multiplicative inverse.

(V) Again commutative law hold good in G being a subset of the set of complex numbers.

Hence G is an abelian group.

2nd part:→ G is a cyclic group generated by 'i'

i.e, $G = \langle i \rangle$, For,

$$1 \in G \Rightarrow 1 = i^4$$
$$-1 \in G \Rightarrow -1 = i^2$$
$$i \in G \Rightarrow i = i^1$$
$$-i \in G \Rightarrow -i = i^3.$$

Scanned with CamScanner

2022 :→ Let $\mathbb{Z}$ be the set of integers. We define $*$ on $\mathbb{Z}$ as follows:–

$a*b = a+b+ab, \ \forall a,b \in \mathbb{Z}$

Examine if $(\mathbb{Z},*)$ is a group.

Solⁿ:- Given that,

Theorem :- In a group G, prove that

(i) Cancellation law hold good in G.

(ii) The inverse of any element of G is unique.

(iii) The identity elt. of G is unique.

(iv) $(a^{-1})^{-1} = a, \forall a \in G$

(v) $(ab)^{-1} = b^{-1}a^{-1}, \forall a, b \in G$.

(vi) The equation $ax = b$ has a unique solution in G, where $a, b \in G$.

Proof :- Given that G is a group.

(i) Let $a, x, y \in G$ such that,

$$ax = ay$$
$$\Rightarrow a^{-1}(ax) = a^{-1}(ay)$$
$$\Rightarrow (a^{-1}a)x = (a^{-1}a)y \quad (\text{by asso. law})$$
$$\Rightarrow ex = ey \quad [aa^{-1} = a^{-1}a = e]$$
$$\Rightarrow x = y$$

$\therefore ax = ay \Rightarrow x = y$.

This shows that left cancellation law hold good in G.