Similarly we can show that Right-cancellation law hold good in G.

Hence cancellation law hold good in G.

(ii). Let x be any elt. of G.

We now show that inverse of x is unique.

If possible, let x_1 & x_2 be two inverse of x.

Then –

$$xx_1 = x_1x = e \quad \text{——(1)}$$

$$\& \; xx_2 = x_2x = e \quad \text{——(2)}$$

$\therefore$ From (1) & (2) we get–

$$xx_1 = xx_2$$

$$\Rightarrow x_1 = x_2 \text{ (by left cancellation law).}$$

This shows that the inverse of x is unique.

Hence every elt. of G has its unique inverse.

(iii). If possible, let e_1 and e_2 be two identity element of G.

Since e_1 is the identity element of G.

$$\therefore xe_1 = e_1x = x, \quad \forall x \in G.$$

$$\Rightarrow e_2e_1 = e_1e_2 = e_2 \quad \text{——(1)} \quad [\text{Taking } x = e_2].$$

Again since e_2 is the identity elt. of G,

$$\therefore xe_2 = e_2x = x, \quad \forall x \in G$$

$$\Rightarrow e_1e_2 = e_2e_1 = e_1 \quad \text{—(2)} \quad [\text{Taking } x = e_1].$$

$\therefore$ From (1) & (11) we get–

$$e_1 = e_2$$

This shows that the identity elt. of G is unique.

(iv) Since, $a \in G$.

$$\therefore a^{-1} \in G \text{ and } (a^{-1})^{-1} \in G.$$

Then,

$$aa^{-1} = a^{-1}a = e.$$

$$\& \; a^{-1}(a^{-1})^{-1} = (a^{-1})^{-1}a^{-1} = e.$$

P.T.O.

Now,

$$aa^{-1}=e$$

$$\Rightarrow (aa^{-1})(a^{-1})^{-1} = e(a^{-1})^{-1}$$

$$\Rightarrow a\{a^{-1}(a^{-1})^{-1}\} = (a^{-1})^{-1} \quad (\text{by asso.})$$

$$\Rightarrow ae = (a^{-1})^{-1}$$

$$\Rightarrow a = (a^{-1})^{-1}$$

$$\Rightarrow (a^{-1})^{-1} = a$$

Proved

(V) To show $(ab)^{-1} = b^{-1}a^{-1}$.

Since $a, b \in G$

$$\Rightarrow a^{-1}, b^{-1} \in G.$$

$$\therefore aa^{-1} = a^{-1}a = e$$

$$\& \; bb^{-1} = b^{-1}b = e.$$

$\boxed{xx_1 = x_1x = e \Rightarrow x_1 = x^{-1}}$

$(\because ab)(b^{-1}a^{-1}) = (b^{-1}a^{-1})(ab) = e$

Now, $(ab)(b^{-1}a^{-1}) = a(bb^{-1})a^{-1} = aea^{-1} = (ae)a^{-1} = aa^{-1} = e$

& $(b^{-1}a^{-1})(ab) = b^{-1}(a^{-1}a)b = b^{-1}eb = b^{-1}(eb) = bb^{-1} = e$

$$\therefore (ab) = (b^{-1}a^{-1}) = (b^{-1}a^{-1})(ab) = e$$

$$\Rightarrow b^{-1}a^{-1} = (ab)^{-1}$$ Proved

$\boxed{xx_1 = x_1x = e \Rightarrow x_1 = x^{-1}}$

(VI) The given equation is

$$ax = b$$

$$\Rightarrow a^{-1}(ax) = a^{-1}b$$

$$\Rightarrow (a^{-1}a)x = a^{-1}b$$

$$\Rightarrow ex = a^{-1}b$$

$$\Rightarrow x = a^{-1}b$$

Since, $a, b \in G$

$$\Rightarrow a^{-1}, b \in G$$

$$\Rightarrow a^{-1}b \in G$$

$$\therefore x = a^{-1}b \in G$$

This shows that the eqn $ax=b$ has soln in G.

Date

Uniqueness :-

If possible, let x_1 and x_2 be two soln of the eqn (1), Then,

$ax_1 = b$

and $ax_2 = b$.

$\therefore ax_1 = ax_2$

$\Rightarrow x_1 = x_2$ [by left cancellation law].

This shows that the given eqn. has a unique soln. *

Finite group and infinite group.

Finite group :-

A group 'G' is said to be a finite group if the set 'G' is finite.

Otherwise it is said to be infinite group if the set 'G' is infinite.

Order of a group :→

Let 'G' be a finite group then the no. of distinct element of 'G' is said to be the order of 'G' and is denoted by $o(G)$.

$\therefore o(G) =$ no. of distinct element of G.

eg → $G = \{1, \omega, \omega^2\}$ is a group w.r.to multiplication.

$\therefore o(G) = 3$.

$G_1 = \{1, -1, i, -i\}$ is a group w.r.to multiplication

$\therefore o(G) = 4$.

Order of an element :→ Let 'a' be any element of a group G. Then the least positive (+ve) integer 'm' is said to be the order of 'a' if $a^m = e$.

$\therefore o(a) = m \Leftrightarrow a^m = e$, where 'm' is the least +ve integer.