(iii) clearly $N = Ne \in G/N$.

We now have,

$$(Nx)N = (Nx)(Ne) = Nxe = Nx$$

$$\& \ N(Nx) = (Ne)(Nx) = Nex = Nx$$

$$\therefore (Nx)N = N(Nx) = Nx, \ \forall Nx \in G/N.$$

This shows that N is the identity element of G/N.

(iv) Let $Nx \in G/N$

Then, $x \in G$

$\Rightarrow x^{-1} \in G$

$\Rightarrow Nx^{-1} \in G/N$

$$\therefore (Nx)(Nx^{-1}) = Nxx^{-1} = Ne = N$$

$$\& \ (Nx^{-1})(Nx) = Nx^{-1}x = Ne = N.$$

$$\therefore (Nx)(Nx^{-1}) = (Nx^{-1})(Nx) = N.$$

This shows that Nx^{-1} is the inverse of Nx.

Hence G/N is a group.

Imp. 106
15/12/09

Q:→ Define a quotient group (or factor group). Prove that every quotient of group of an abelian group is abelian.

Soln:→ Quotient group:→ Let N be a normal subgroup of a group G and G/N be the set of all left co-sets of N in G ie,

$$G/N = \{Nx \mid x \in G\}.$$

Then G/N is a group w.r.to the binary operation defined by,

$$(Nx)(Ny) = Nxy, \ \forall Nx, Ny \in G/N.$$

What we called quotient group.

2nd part: Let G be an abelian group and N be a normal subgroup of G. Then

$$G/N = \{Nx \mid x \in G\}.$$

is a quotient group.

We are to show that G/N is also abelian.

Let $Nx, Ny \in G/N$. Then $x, y \in G$.

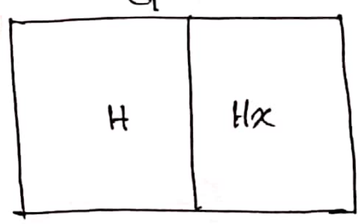
$$\therefore (Nx)(Ny) = Nxy$$
$$= Nyx \quad [\because x, y \in G \Rightarrow xy = yx]$$
$$= (Ny)(Nx)$$

$$\therefore (Nx)(Ny) = (Ny)(Nx), \ \forall Nx, Ny \in G/N.$$

This shows that G/N is an abelian group.

Q: Prove that the subgroup H of a group G of index two is a normal subgroup.

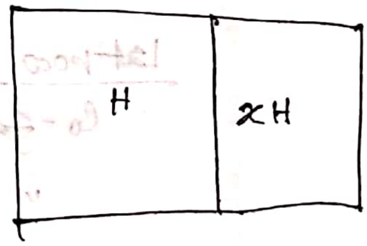
G



Proof: Since H is a subgroup of G of index two, Therefore the no. of distinct right coset or left coset must be two.

Since, H is itself is a left coset as well as right co-set, Therefore ~~tof~~ let xH and Hx be the other two left co-set and right co-set of H in G.

G



$$\therefore G = H \cup Hx \ \& \ G = H \cup xH.$$

$$\therefore H \cup Hx = H \cup xH$$

Since, $H \cap Hx = \phi$ & $H \cap xH = \phi$,

$$\therefore Hx = xH, \ \forall x \in G$$

This shows that H is a normal subgroup of G.