V_2) Again we have,

$$\alpha(x+y) = \alpha\big((x_1, x_2, \cdots x_n) + (y_1, y_2, \cdots y_n)\big)$$
$$= \alpha(x_1+y_1, x_2+y_2, \cdots, x_n+y_n)$$
$$= \big(\alpha(x_1+y_1), \alpha(x_2+y_2), \cdots, \alpha(x_n+y_n)\big)$$

[by scalar multiplication]

$$= (\alpha x_1+\alpha y_1, \alpha x_2+\alpha y_2, \cdots, \alpha x_n+\alpha y_n)$$
$$= (\alpha x_1, \alpha x_2, \cdots, \alpha x_n) + (\alpha y_1, \alpha y_2, \cdots \alpha y_n)$$
$$= \alpha(x_1, x_2, \cdots, x_n) + \alpha(y_1, y_2, \cdots y_n)$$
$$= \alpha x + \alpha y$$

$\therefore \alpha(x+y) = \alpha x + \alpha y, \forall x, y \in \mathbb{R}^n \;\&\; \forall \alpha \in \mathbb{R}$.

V_3) Also we have,

$$(\alpha+\beta)x = (\alpha+\beta)(x_1, x_2, \cdots, x_n)$$
$$= \big((\alpha+\beta)x_1, (\alpha+\beta)x_2, \cdots, (\alpha+\beta)x_n\big)$$
$$= (\alpha x_1+\beta x_1, \alpha x_2+\beta x_2, \cdots, \alpha x_n+\beta x_n)$$
$$= (\alpha x_1, \alpha x_2, \cdots, \alpha x_n) + (\beta x_1, \beta x_2, \cdots, \beta x_n)$$
$$= \alpha(x_1, x_2, \cdots x_n) + \beta(x_1, x_2, \cdots x_n)$$
$$= \alpha x + \beta x$$

$\therefore (\alpha+\beta)x = \alpha x + \beta x, \forall \alpha, \beta \in \mathbb{R}, \forall x \in \mathbb{R}^n$.

V_4) We have,

$$(\alpha\beta)x = (\alpha\beta)(x_1, x_2, \cdots, x_n)$$
$$= \big((\alpha\beta)x_1, (\alpha\beta)x_2, \cdots (\alpha\beta)x_n\big)$$
$$= \big(\alpha(\beta x_1), \alpha(\beta x_2), \cdots \alpha(\beta x_n)\big)$$

[by asso. in $\mathbb{R}$]

$$= \alpha(\beta x_1, \beta x_2, \cdots \beta x_n)$$
$$= \alpha\big(\beta(x_1, x_2, \cdots x_n)\big)$$
$$= \alpha(\beta x)$$

$\therefore (\alpha\beta)x = \alpha(\beta x), \forall \alpha, \beta \in \mathbb{R} \;\&\; \forall x \in \mathbb{R}^n$

P.T.O

$V_5)$ We have,

$$1.x = 1(x_1, x_2, \ldots x_n)$$
$$= (1x_1, 1x_2, \ldots 1x_n)$$
$$= (x_1, x_2 \ldots\ldots x_n)$$
$$= x$$

$\therefore 1.x = x \; ; \forall x \in \mathbb{R}^n$

where 1 is the multiplicative identity of the field $\mathbb{R}$.

· Hence, $\mathbb{R}^n(\mathbb{R})$ is a vector space. //

'08 Q. Let $V = \{x = (\alpha \beta) \mid \alpha, \beta \in \mathbb{R}\}$. Examine whether V is a * vector space over the field of real numbers $\mathbb{R}$ or not w.r.to point wise addition and scalar multiplication

$$(\alpha, \beta) + (\gamma, \delta) = (\alpha + \gamma, \beta + \delta)$$
$$\& \quad a(\alpha, \beta) = (a\alpha, a\beta)$$

Solⁿ Here defined vector addition and scalar multiplication by–

1. Vector addition –

$$\therefore x + y = (\alpha, \beta) + (\gamma, \delta)$$
$$= (\alpha + \gamma, \beta + \delta), \forall x = (\alpha, \beta), y = (\gamma, \delta) \in V$$

2. Scalar multiplication –

$$ax = a(\alpha, \beta)$$
$$= (a\alpha, a\beta); \forall x = (\alpha, \beta) \in V \;\&\; a \in \mathbb{R}$$

We now show that $V(\mathbb{R})$ is a vector space

$V_1)$ (i) From the defination it is clear that

$$x + y \in V, \forall x, y \in V$$

(ii) $x + (y + z) = (\alpha, \beta) + \{(\gamma, \delta) + (u, v)\}$

$$= (\alpha \beta) + (\gamma + u, \delta + v)$$
$$= (\alpha + (\gamma + u), \beta + (\delta + v))$$
$$= ((\alpha + \delta) + u, (\beta + \delta) + v)$$
$$= (\alpha + \delta, \beta + \delta) + (u, v)$$
$$= ((\alpha, \beta) + (\gamma, \delta)) + (u, v)$$
$$= (x + y) + z$$

$\therefore x + (y + z) = (x + y) + z \; ; \forall x, y, z \in V.$

P.T.O

(iii) Clearly., $\bar{0}=(0,0)\in V$

We now have,

$$x+\bar{0}=(\alpha,\beta)+(0,0)$$
$$=(\alpha+0,\beta+0)$$
$$=(\alpha,\beta)$$
$$=x$$

Similarly, we now show that

$$\bar{0}+x=x$$
$$\therefore x+\bar{0}=\bar{0}+x=x,\ \forall\ x\in V$$

(iv) For any $x=(\alpha,\beta)\in V$, $\exists\ -x=(-\alpha,-\beta)\in V$

We have, $x+(-x)=(\alpha,\beta)+(-\alpha,-\beta)$
$$=(\alpha+(-\alpha),\beta+(-\beta))$$
$$=(0,0)$$
$$=\bar{0}$$

Similarly we can show that

$$(-x)+x=\bar{0}$$
$$\therefore x+(-x)=(-x)+x=\bar{0}$$

This shows that '$-x$' is the ~~identity~~ additive inverse of x.

(v) Again we have;

$$x+y=(\alpha,\beta)+(\gamma,\delta)$$
$$=(\alpha+\gamma,\beta+\delta)$$
$$=(\gamma+\alpha,\delta+\beta)$$ [By commutative law in $\mathbb{R}$]
$$=(\gamma,\delta)+(\alpha,\beta)$$
$$=y+x$$
$$\therefore x+y=y+x,\ \forall\ x,y\in V$$

Hence, V is an additive abelian group.

V_2> we have —

$$a(x+y)=a\big((\alpha,\beta)+(\gamma,\delta)\big)$$
$$=a(\alpha+\gamma,\beta+\delta)$$
$$=(a(\alpha+\gamma),a(\beta+\delta))$$
$$=((a\alpha+a\gamma),(a\beta+a\delta))$$
$$=((a\alpha+a\gamma),(a\beta+a\delta))$$
$$=(a\alpha,a\beta)+(a\gamma,a\delta)$$

P.T.O