(ii)

$\because 0 \in F$

$\therefore 0+0=0$

$\Rightarrow (0+0).\bar{x} = 0.\bar{x}$

$\Rightarrow 0.\bar{x} + 0.\bar{x} = 0.\bar{x}$

$\Rightarrow 0.\bar{x} + 0.\bar{x} = 0.\bar{x} + \bar{0}$

$\Rightarrow 0.\bar{x} = \bar{0}$ [by left cancelation law]

$\therefore 0.\bar{x} = \bar{0}, \forall \bar{x} \in X \ \& \ 0 \in F$.

(iii)

We know that

$\cancel{0.\bar{x} = \bar{0}}$

$\cancel{\Rightarrow (\alpha + (-\alpha))\bar{x} = \bar{0}}$

$\cancel{\Rightarrow \alpha\bar{x} + (-\alpha)\bar{x} = \bar{0}}$

$\bar{x} + (-\bar{x}) = \bar{0}$ $\quad \alpha.\bar{0} = \bar{0}$

$\cancel{\Rightarrow \alpha(\bar{x} + (-\bar{x}))}$ $\Rightarrow \alpha.(\bar{x} + (-\bar{x})) = \bar{0}$

$\Rightarrow \alpha\bar{x} + \alpha(-\bar{x}) = \bar{0}$

This shows that $\alpha(-\bar{x})$ $\cancel{(-\alpha)\bar{x}}$ is the additive inverse of $\alpha\bar{x}$. i.e

$\alpha(-\bar{x}) \cancel{(-\alpha)\bar{x}} = -(\alpha\bar{x}) \rightarrow ①$

But, $(-\alpha)\bar{x} = -(\alpha\bar{x}) \rightarrow ②$

Hence from ① & ② we get

$(-\alpha)\bar{x} = \alpha(-\bar{x}) = -(\alpha\bar{x})$ Proved

(iv)

We know that,

$(-\alpha)\bar{x} = -(\alpha\bar{x})$

Putting $\alpha = 1$, we get

$(-1)\bar{x} = -(1.\bar{x})$

$\Rightarrow (-1)\bar{x} = -\bar{x}$

Proved

(v) Given that,

$\alpha\bar{x} = \bar{0} \rightarrow ①$

To show either $\alpha = 0$ or $\bar{x} = \bar{0}$

Let, $\alpha \neq 0$. To show $\bar{x} = \bar{0}$

Since $\alpha \neq 0$, α^{-1} exists & $\alpha\alpha^{-1} = \alpha^{-1}\alpha = 1$.

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Now, ① $\Rightarrow \alpha\bar{x}=\bar{0}$

$\Rightarrow \alpha^{-1}(\alpha\bar{x})=\alpha^{-1}\bar{0}$

$\Rightarrow (\alpha^{-1}\alpha)\bar{x}=\bar{0}$

$\Rightarrow 1\bar{x}=\bar{0}$

$\Rightarrow \bar{x}=\bar{0}$

Conversly, let $\bar{x}\neq\bar{0}$, To show $\alpha=0$

If possible, let $\alpha\neq 0$

Then, α^{-1} exists and $\alpha\alpha^{-1}=\alpha^{-1}\alpha=1$

Now, ① $\Rightarrow \alpha\bar{x}=\bar{0}$

$\Rightarrow \alpha^{-1}(\alpha\bar{x})=\alpha^{-1}\bar{0}$

$\Rightarrow (\alpha^{-1}\alpha)\bar{x}=\bar{0}$

$\Rightarrow 1.\bar{x}=\bar{0}$

$\Rightarrow \bar{x}=\bar{0}$.

which contradicts to the fact that $\bar{x}\neq\bar{0}$.

$\therefore \alpha=0$

$\therefore \alpha\bar{x}=\bar{0}\Rightarrow$ either $\bar{x}=\bar{0}$ or $\alpha=0$//.

—o—

Dt. 7.9.08

Defination of vector sub-space :- A non-empty subset W of a vector space V(F) is said to be a vector subspace of the vector space V(F) if W is itself is a vector space over the field F w.r. to the vector addition and scalar multiplication in V(F).

Note:- If V(F) is a vector space then it has at least two subspace namely {0} and the vector space V(F) itself. These two subspaces are said to be a trivial subspaces of V(F). Excluding these two subspaces, the other sub-spaces are said to be a non-trivial subspace of V(F).

P.T.O

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Span of a set or generating set :-

Let $V(F)$ be a vector space and $S \subseteq V$.
Then the smallest vector subspace containing the set S is said to be a subspace generated by the set 'S' or span by the set 'S' and it is denoted by $\langle S \rangle$.

$\therefore \langle S \rangle =$ The smallest vector subspace containing the The set S.

If S is finite, then $\langle S \rangle$ is said to be a finitely generated subspace of $V(F)$.

Note:- We know that $V(F)$ is a subspace of a vector space $V(F)$ itself. Thus if S is a subset of V such that $\langle S \rangle = V(F)$. i.e $V(F)$ is the only subspace containing the set S, then the vector space $V(F)$ is said to be generated by the set S or span by the set S.

If S is finite and $\langle S \rangle = V(F)$, then the vector space $V(F)$ is said to be finitely generated vector space.

1. Theorem:- Prove that a necessary and sufficient condition for a non-empty subset W of a vector space $V(F)$ to be a subspace is that $\alpha x + \beta y \in W$, $\forall \alpha, \beta \in F$ & $\forall x, y \in W$.

Proof:- Given that W is a non-empty subset of a vector space $V(F)$. we are to show that W is a subspace of $V(F) \Leftrightarrow \alpha x + \beta y \in W$, $\forall \alpha, \beta \in F$ & $\forall x, y \in W$.

First let W be a subspace of $V(F)$.
Then for any $x, y \in W$ & $\alpha, \beta \in F$, we have -

$$\alpha x, \beta y \in W$$

$$\Rightarrow \alpha x + \beta y \in W \quad [\because W \text{ is a subspace of } V(F)$$

$$\therefore \alpha x + \beta y \in W, \ \forall x, y \in W \ \& \ \forall \alpha, \beta \in F$$

This shows that the condition is necessary.

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